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13. ABSTRACT (Maximum 200 words)  I proposed to use ideas from classical stochastic search algorithms to design a quantum computing algorithm for optimization which would tolerate quantum decoherence. I applied ideas from classical annealing algorithms to quantum optimization. I applied the Redfield theory for spin systems to model decoherence in a quantum computer. I designed a simple quantum computing cell which could be analyzed in terms of the Redfield equation. And I showed how a large number of such cells could be connected to perform a quantum version of simulated annealing for combinatorial optimization. Unfortunately, I did not succeed in effectively and correctly applying the proposed quantum annealing computer to the <i>Min Cut</i> combinatorial optimization problem I chose to attack.				
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# Final Progress Report on Quantum Optimization

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## 1 Statement of problem

I proposed to attack the problem of extreme sensitivity of quantum computing to decoherence by designing a new kind of quantum computer that could tolerate some decoherence by exploiting ideas from classical iterative optimization algorithms (such as steepest descent) and from stochastic search algorithms (such as simulated annealing).

QC depends delicately on the coherent superposition of so many quantum states that any computation may be too sensitive to imperfections in the implementation of program steps and maybe too sensitive to interactions with the external environment.

However some classical algorithms such as stochastic descent algorithms tolerate roundoff errors and even a finite number of large disturbances, since they are designed to gradually reduce any current error regardless whether its origin is poor initialization or perturbations in each step.

## 2 Results

### 2.1 Quantum version of stochastic search algorithm

I applied ideas from classical annealing algorithms[1, 2] to quantum computation of a minimum for the first time as follows.

Define  $dU = \prod_m U_m$ , so that for some  $H_s$ ,

$$dU = e^{-iH_s dt},$$

then execute the following algorithm:

*REPEAT*

Apply sequence of  $U_m = U(i_m, j_m, k_m, \theta_m)$   
 3 bits at a time, with each  $|\theta_m| \ll \pi$ ,  
 Interacting with environment

*UNTIL* measurement

Execution obeys  $i\hbar\dot{\psi}_s = H_s(t)\psi_s$ , so that the state of the computation is

$$\psi_s(t) = e^{iH_s t} \psi_s(0).$$

The program is  $H_s$ .

### 2.2 Redfield theory to analyze decoherence

I applied the Redfield theory for spin systems[3] to model decoherence in a quantum computer, for the first time as follows. The density matrix

$$\rho(t) = \sum_x p_x |x\rangle\langle x|$$

in the interaction representation

$$\rho^I \equiv e^{i(H_s + H_b)t/\hbar} \rho e^{-i(H_s + H_b)t/\hbar}$$

obeys the linear equations

$$\begin{aligned} \frac{d}{dt} \rho_{s\alpha\alpha'}^I(t) &= \\ &- \sum_{\beta\beta'} e^{\frac{i}{\hbar}(\epsilon_\alpha - \epsilon_\beta - \epsilon_{\alpha'} + \epsilon_{\beta'})} \\ &R_{\alpha\alpha'\beta\beta'} (\rho_{s\beta\beta'}^I - \langle \rho_{s\beta\beta'}^I \rangle_T) \end{aligned}$$

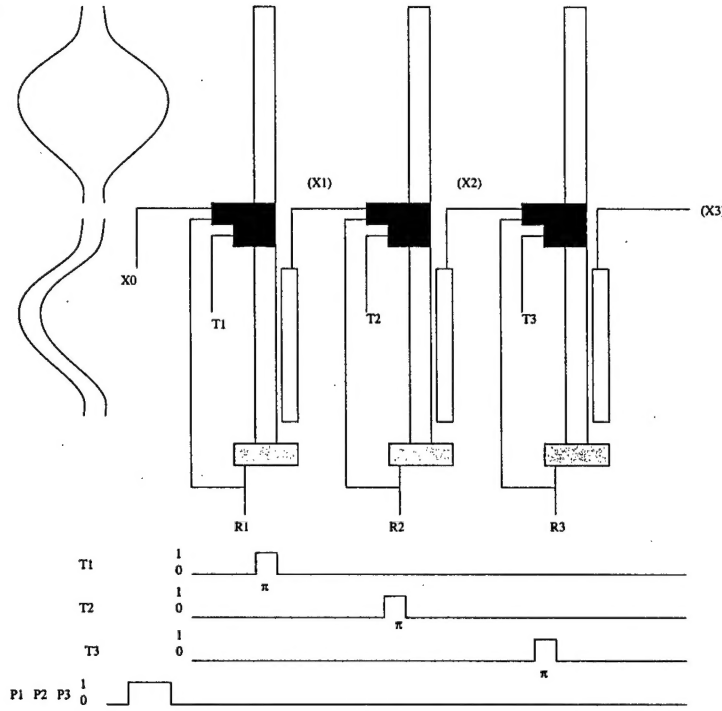


Figure 1: Quantum Shift Register

for finite  $T$  [4, p174].

### 2.3 Quantum computational cell

I designed a simple quantum computing cell which could be analyzed in terms of the Redfield equation as shown in Figure 1.

### 2.4 Quantum architecture

And I showed how a large number of such cells could be connected to perform a quantum version of simulated annealing for combinatorial optimization as shown in Figure 2.

Unfortunately, I did not succeed in effectively and correctly applying the proposed quantum annealing computer to the *Min Cut* combinatorial optimization problem I chose to attack.

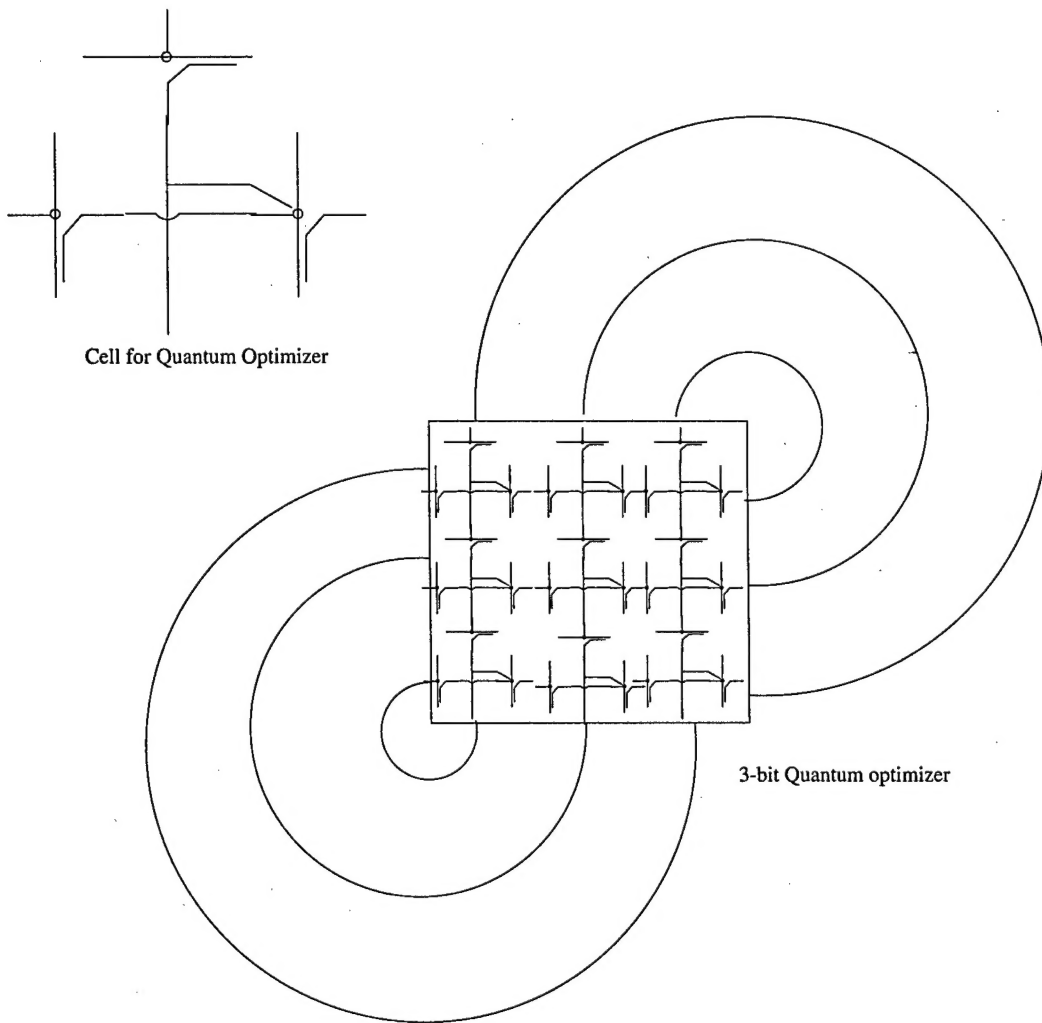


Figure 2: Architecture of a Quantum Optimizer

## References

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